Name:______________________________ Date:____________________________

SOLUTION for Pre-Calculus 11 HW 4.5 Discriminant Nature of the Roots $D=b^2-4ac$

1. Determine the nature of the roots [ie: Determine how many x-intercepts each quadratic equation has]

a) $x^2+5x+6=0$ To find the nature of the roots, just use the discriminant formula: $D=b^2-4ac$ $D=25-4(1)(6)$ $D=1$ There are two distinct real roots	b) $12x^2+7x-3=0$ $D=b^2-4ac$ $D=49-4(12)(-3)$ $D=49+144=193>0$ There are two distinct real roots	c) $-2x^2-7x+5=0$ $D=b^2-4ac$ $D=49-4(-2)(5)$ $D=89>0$ There are two distinct roots
d) $4x^2=13x-8$ $4x^2-13x+8=0$ $D=b^2-4ac$ $D=169-4(4)(8)$ $D=41>0$	e) $x(7-8x)=10$ $7x-8x^2=10$ $0=8x^2-7x+10$ $D=b^2-4ac$ $D=49-4(8)(10)$ $D=-271<0$ No real roots	f) $x(x+2)=6-(x-3)(2x+1)$ $x^2+2x=6-(2x^2-5x-3)$ $x^2+2x=6-2x^2+5x+3$ $3x^2-3x-9=0$ $D=b^2-4ac$ $D=9-4(3)(-9)$ $D=117>0$ There are two distinct roots

2. Solve each of the following inequalities:

a) $x^2<16$ $-4 < x < 4$	b) $x^2-25>0$ $x^2>25$ $x<-5$ *or* $5<x$	c) $x(3-x)<0$ $x(3-x)=0$ $x=0, x=3$ $x(3-x)<0$ $x<0$ *or* $3<x$

3. Determine the value of "k" so that the equation has two equal roots:

a) $x^2+kx+25=0$ To have two equal roots, the discriminant must be equal to zero. $D=b^2-4ac$ $k^2-4(1)(25)=0$ $k^2=100$ $k=\pm 10$	b) $kx^2+4x+1=0$ $D=b^2-4ac$ $16-4(k)(1)=0$ $16=4k$ $4=k$	c) $0.5x^2+3kx+(3k-4)=0$ $D=b^2-4ac$ $(3k)^2-4(0.5)(3k-4)=0$ $9k^2-2(3k-4)=0$ $9k^2-6k+8=0$ $(3k-4)(3k+2)=0$ $k=\frac{4}{3}$ *or* $k=\frac{-2}{3}$

4. Determine the value of "k" so that the equation has two different roots:

a) $x^2 - kx + 12 = 0$ To have two distinct roots, the discriminant must be greater than 0. $b^2 - 4ac > 0$ $k^2 - 4(12) > 0$ $k^2 - 48 > 0$ $k < -4\sqrt{3} \quad or \quad 4\sqrt{3} < k$ Draw a number line and use test points:	b) $kx^2 - kx + 1 = 0$ $b^2 - 4ac > 0$ $k^2 - 4(k) > 0$ $k(k-4) > 0$ $k < 0 \quad or \quad 4 < k$	c) $x^2 - 4kx + (5k-6) = 0$ $b^2 - 4ac > 0$ $(-4k)^2 - 4(1)(5k-6) > 0$ $16k^2 - 20k + 24 > 0$ $4(k^2 - 5k + 6) > 0$ $4(k-2)(k-3) > 0$ $k < -2 \quad or \quad 3 < k$

5. Determine the value of "k" so that the equation has no real roots:

a) $x^2 - kx - 24 = 0$ To have no real roots, the discriminant must be less than 0 $b^2 - 4ac < 0$ $k^2 - 4(-24) < 0$ $k^2 + 96 < 0$ The left side is always positive, because k^2 is always positive. SO, no matter what "k", the equation will always have 2 distinct roots	b) $kx^2 - kx + 8 = 0$ $b^2 - 4ac < 0$ $k^2 - 4(k)(8) < 0$ $k^2 - 32k < 0$ $k(k-32) < 0$ $0 < k < 32$ if "K' is between 0 and 32, the equation will not have any roots	c) $x^2 - 3kx - (3k-8) = 0$ $x^2 - 3kx - 3k + 8 = 0$ $b^2 - 4ac < 0$ $(-3k)^2 - 4(-3k+8) < 0$ $9k^2 + 12k - 32 < 0$ $(3k+8)(3k-4) < 0$ $\frac{-8}{3} < k < \frac{4}{3}$

6. In order for a quadratic function to be factorable, what value must the discriminant be equal to? Explain:

This is the quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. In order for a quadratic equation to be factorable, both roots must be either an integer or a fraction. Can't have a radical. So that means the discriminant $b^2 - 4ac$ needs to be either 0 or a perfect square.

If the quadratic equation $(x-2)^2 + k = 0$ has two distinct real roots, then what is the range of "k"? (Multiple choice, circle one) Justify your answer.

a) $k > 2$ b) $k < 0$ c) $k \le 0$ d) $k \le 4$

$x^2 - 4x + 4 + k = 0$

$16 - 4(1)(4+k) > 0$

$16 - 4(4+k)$

$16 - 16 - 4k > 0$

$-4k > 0$

$k < 0$

So as long as k<0, the quadratic equation will have two distinct roots